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A Principle for Network Science

by Bruce J. West and Paolo Grigolini

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A Principle for Network Science

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14. ABSTRACT

Network Science forms one of the most challenging areas of modern research overarching the traditional scientific disciplines. Of particular importance is the manner in which information is shuttled back and forth between complex networks, and whether or not there exists general principles that guide the flow of information. Herein, we identify Wiener's rule, which conjectures how information is transferred in an information-dominated process. Moreover, we show that this rule is a consequence of a Principle of Complexity Management (PCM) that determines the efficiency of information exchange between complex networks and has been proven in a mathematically rigorous manner in the past year. The applications of PCM to the phenomena of habituation, forgetting, and decision making made in the past year are reviewed.

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1. Introduction

Physics formulates universal laws and principles involving phenomena in the physical world, laws that capture a vast array of experimental results with a minimum of assumptions. The clockwork universe of Newton was a satisfactory explanation of the world for a large fraction of civilized society. However, others could see that this was not adequate for characterizing dynamics in the social or life sciences.

The soft sciences of psychology and sociology lack the universality of physics: the conservation of energy, symmetry principles, and the laws of thermodynamics have no analogs in the soft sciences. This lack of universality arises, in part, because of the lack of appropriate metrics. In a physical interaction, there is always the exchange of something tangible: momentum, energy, matter, or the element of an appropriate field such as a photon or phonon. However, in a social interaction, a far more elusive quantity is exchanged and that is information, whose dynamics remain elusive in a social setting. Herein, we review of the insights provided by our investigations into the information exchange between complex networks (*1*).

In section 2, we discuss Wiener's rule for the flow of information between networks in which the information is transferred from high to low. Unlike physical systems in which forces are produced by energy gradients between networks, in information systems, forces are produced by information gradients between networks. This speculation of Wiener (*2*) is shown to be a consequence of a more general principle involving the relative complexity measures between two interacting networks and is called the Principle of Complexity Management (PCM). We gave an empirical justification for PCM (*1*) and subsequently were able to prove it in a mathematically rigorous way (*3*).

In section 3, we demonstrate how PCM explains a variety of the social and psychological phenomena including the simple learning process of habituation, how we forget, and how rational the decisions we make are. Section 4 provides a brief summary.

2. Wiener's Rule

A frequently asked question is: Can't the methods of non-equilibrium statistical physics be applied to the description of the dynamics of complex social and psychological networks? Various answers to this question have been given, but one of the most interesting answers was given by the mathematician Norbert Wiener in the middle of the last century. Wiener was the first scientist to identify the complexity associated with communication and control in living organisms, machines and their interaction. He proposed mathematical methods for the description of their dynamics with the creation of *Cybernetics*. The depth of his insight is revealed in his intuitive observation (*2*):

> "We have a system of high energy coupled to a message low in energy, but extremely high in amount of information, i.e., of great negative entropy. This is unlike the usual interaction in thermodynamics, where all the coupled systems enjoy high entropy. But it may happen in the development of such a system that the internal coupling causes the information, or negative entropy, to pass from the part at low energy to the part at high energy so as to organize a system of vastly greater energy than that of the present instantaneous input."

From this quote we conclude that the Second Law of Thermodynamics can apparently be subverted when two complex networks talk to one another if the information content of one far exceeds the information content of the other. It is the network with the greater information, but lesser energy that organizes the network of greater energy but lesser information. This is shown schematically in figure 1 where the Second Law of Thermodynamics is identified by an energy-dominated process in the upper panel. In the lower panel, a network with high energy but low information is depicted, which is unlike the thermodynamic situation. In this latter case, the network with the greater information can control the network with the lesser information, even though the latter may have the greater energy. This is an information-dominated process.

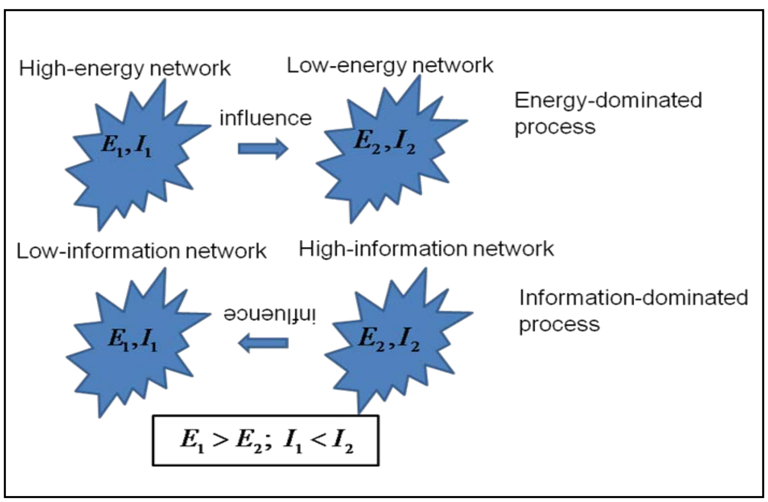

Figure 1. Wiener's rule: The upper panel denotes the familiar thermodynamic situation of an energy-dominated process, and the lower panel depicts the counter-intuitive information dominated process.

The transfer of influence from a complex network high in information to one low in information is the first attempt at a universal principle of network science and represents what we call Wiener's rule. Of course, Wiener was speculating, giving the audience the benefit of his experience. That insight was not vindicated for another half century (*3*) and required the generalization of a number of basic concepts from statistical physics (*4, 5*).

One measure of the information content of a network is provided by the probability density most often used in the determination of the negative entropy that Schrödinger dubbed negentropy. An apparently ubiquitous distribution in the description of empirical complex networks is the hyperbolic, which denotes the survival probability as

$$\Psi(t) = \frac{T^{\mu-1}}{\left(T+t\right)^{\mu-1}} \tag{1}$$

that asymptotically becomes an inverse power law. We could list a hundred natural, social, and medical phenomena that display this behavior but a few exemplars should suffice, such as blackout intervals on the North American power grid, physiologic processes such as breathing and heart beats, solar flare eruptions on the sun, and the time intervals between earthquakes of a given magnitude (*1*, *6*). The average time between events in all the complex webs mentioned can be determined using the probability density $\psi(t) = -d\Psi(t)/dt$, which is

$$\langle t\rangle = \int_0^\infty t\psi\left(t\right)dt = \begin{cases} \dfrac{T}{\mu-2}; & \mu > 2 \text{ ergodic} \\ \infty\,; & \mu < 2 \text{ non-ergodic} \end{cases} \tag{2}$$

It is interesting that when the power-law index is in the interval $2<\mu<3$ the distribution has a finite first moment and the statistics are ergodic, meaning that the time average and ensemble averages yield the same result. However, when the power-law index is $0<\mu<2$ there are no finite integer moments and the ensemble and time averages are not the same, that is, the process is non-ergodic, which is to say that the ensemble and time averages are not the same.

The mathematical/computational verification of Wiener's insight required over half a century and uses the properties of complex networks (*3*). However, in this report, we are interested in making the first applications of Wiener's rule and not in reiterating the underlying mathematics. These details are presented in our most recent book *Complex Webs: Anticipating the Improbable* (*6*). One measure of the information transfer between two complex networks is the cross-correlation between a complex network P and a complex network S being perturbed by P with ε, the strength of the perturbation. For our purposes, it is sufficient to apply the generalized linear response theory (*4*, *5*) we previously developed to the cross-correlation function (*3*):

$$C(t) = \varepsilon\int_0^t R_x(t')\Psi_s(t-t')\Psi_p(t,t')dt'\,. \tag{3}$$

The perturbing complex network P is characterized by the non-stationary autocorrelation function $\Psi_p(t,t')$, which depends separately on the time of the last perturbation *t'* and the time of the measurement *t*. The function $R_s(t')$ is the rate of generating perturbing events at time *t'* within the network being perturbed and is based on renewal theory (*1*). The perturbed network S is characterized by the stationary autocorrelation function $\Psi_s(t-t')$, which depends only on the difference in times from the last perturbation to the measurement.

In figure 2, the asymptotic cross-correlation function normalized to the strength of the perturbation is graphed as a function of the power-law indices of the two networks to form the cross-correlation cube. This cube displays a number of remarkable properties:

1. When the power-law indices are both equal to 2, there is an abrupt jump from zero correlation to perfect consensus.

2. The upper plateau indicates that when P is non-ergodic $1<\mu_p<2$ and S is ergodic $2<\mu_s<3$, there is an information response in which the perturbed network tracks the perturbing network exactly and the information transfer is maximal. The time interval between the non-ergodic perturbing events is much shorter than those between the ergodic events occurring naturally in S. Consequently, the response of S adopts the statistics of P.

3. When P is ergodic $2<\mu_p<3$ and S is non-ergodic $1<\mu_s<2$, there is no response asymptotically and the information transfer is minimal. In this configuration, the time interval between perturbing events is greater than those in the non-ergodic responding network and tends to be swallowed up. How a complex network responds to a perturbation by another complex network is determined by whether or not there is a statistical mismatch in the complexity of the fluctuations in the two networks.

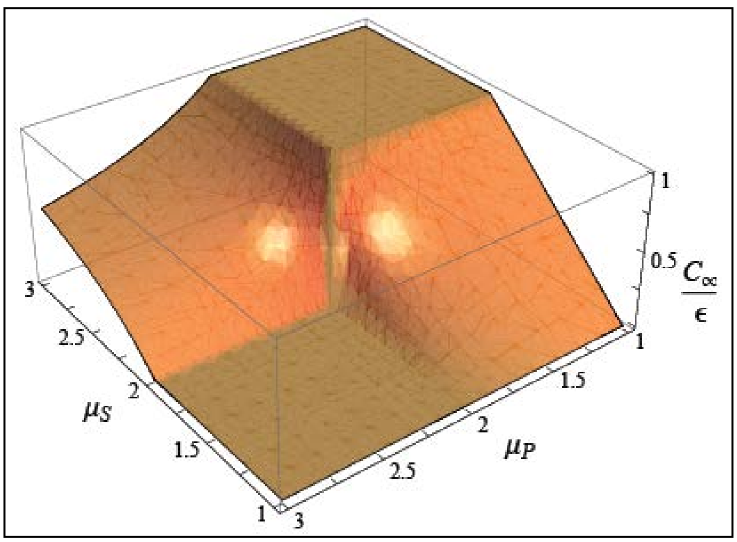

Figure 2. The cross-correlation cube. The asymptotic cross-correlation function defined by equation 3 is graphed as a function of the two power-law indices of the perturbed network S and the perturbing network P.

Wiener's rule describes the influence of the perturbing network outside the lower plateau region of the cross-correlation cube. In all regions except this one, the weak perturbation significantly modifies the properties of the complex network being perturbed. In the upper plateau region, the perturbation by network P actually dominates the properties of the perturbed network S and reorganizes it, just as Wiener predicted. The PCM embodied in the cross-correlation cube therefore subsumes Wiener's rule.

3. From the Cerebral to the Common Place

Let us now turn our attention from the general formalism to some applications of the cross-correlation cube and PCM. We give examples from three of our publications of various ways the human brain functions during the processes of habituation, forgetting, and decision making. Some are applications of Wiener's rule but they are all described by PCM as embodied by the cross-correlation cube, as we shall see.

3.1 Habituation

The first example is that of habituation, which is a ubiquitous and extremely simple form of learning through which animals, including humans, learn to disregard stimuli that are no longer novel, thereby allowing them to attend to new stimuli (*7*). A repetitious stimulus of unvarying amplitude and frequency content induces a response that fades over time since no new information is being presented to the animal. The lack of new information allows the brain to shift its focus from the more to the less familiar, the latter providing new information that may have survival value. However, we know the brain does not habituate to all external stimuli so let us consider two distinct kinds of stimulation: one ergodic $2 \le \mu_p \le 3$ and another non-ergodic $1 \le \mu_p \le 2$. The perturbation in the ergodic regime can be expressed as a decomposition of linear harmonic terms, which allows us to generalize a previously established result for a periodic stimulation of a complex network (*8*). The non-ergodic stimulus can be drawn from a number of sources; here we choose the sequence of splashes from a dripping faucet (*9*) and classical music (*10*).

Consider first the case of highway noise coming in through the window as one lies in bed at night. This noise is typically a broadband, uncorrelated, random process and consequently it is ergodic. Most people habituate to this noise and soon fall asleep. In the present context, it is possible to prove that the response of the brain to the noise fades as an inverse power-law in the following way (*8*):

$$D(t) \propto \frac{1}{t^{|2-\mu_s|}} \quad . \tag{4}$$

The lower plateau II in figure 3 is the parameter region $2 \le \mu_p \le 3$, $1 \le \mu_s \le 2$, where the dynamics of the brain and the external stimulus are asymptotically statistically independent of one another. Brain activity in the non-ergodic regime $1 \le \mu_s \le 2$ is asymptotically unresponsive to ergodic and/or periodic stimuli; the complexity of the neuron network essentially swallows up simple signals through its complex dynamic interactions and the response fades as described by equation 4. In the present context, we used generalized linear response theory to determine the asymptotic suppression of the stimulus to explain the phenomenon of habituation (*8*). But even when the brain is in the ergodic regime $2 \leq \mu_s \leq 3$, its response habituates to a constant value less than the full response value of the upper plateau, but it does not vanish altogether.

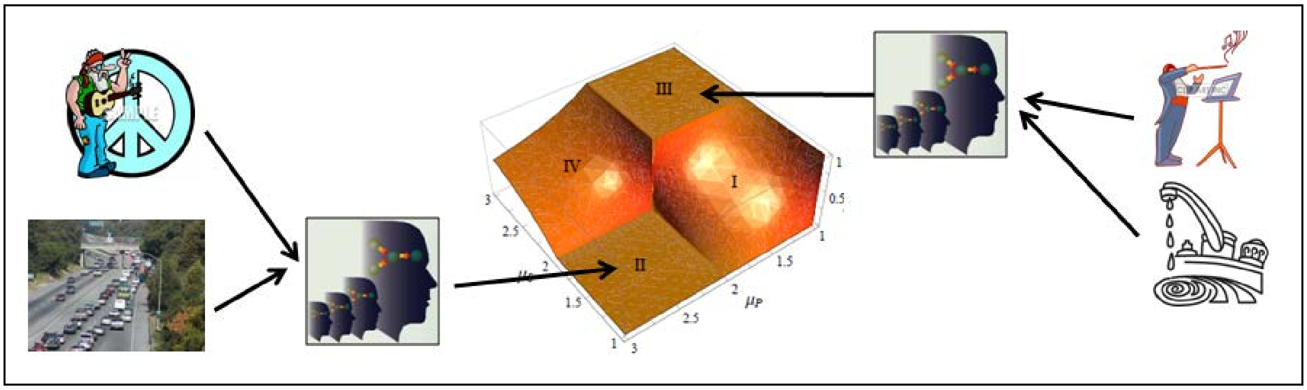

Figure 3. Examples of stimuli that habituate asymptotically and those that are in “resonance” with the complexity of the human brain and do not habituate.

The second stimulus we consider is the sound of splashing water from a leaky faucet. This sequence of water drops can set your teeth on edge and leads to tossing and turning throughout the night. The leaky faucet stimulus has been determined experimentally to have a distribution with a power-law index in the domain $1 \leq \mu_p \leq 2$; it is non-ergodic (*9*). The brain’s response ramps up from 0 to 1 as the index μ_s increases from 1 to 2, as shown in figure 3. Over the interval $2 \leq \mu_s \leq 3$, the response to the intermittent splashes given by the cross-correlation is maximal. This upper plateau III depicts the parameter domain where the brain is ergodic and records the sound of every intermittent drop of water.

It was determined experimentally (*9*) that the intermittent sequence of water drops from a leaky faucet is described by a Lévy stable distribution that is an asymptotically inverse power-law with index $\mu_p=1.73$. Consequently, the power spectral density has an index $\alpha_p = 1.27$, very close to the value where Wiener’s rule predicts the maximum response of the brain to the stimulus.

Of course, it is not just annoying stimuli that refuse to fade away. Classical music has been shown to manifest 1/f behavior (*10*) and resonate with the human brain, leaving strains of melody running through your head long after the music stops. The influence of the more pleasant stimuli also resides on the plateau III of figure 3. West and Deering (*11*) review the occurrence of the 1/f variability in classical western music, classical ragas of India, traditional music of Japan, and jazz, as well as the spatial variability in successful paintings. They speculated that the aesthetic judgments we make regarding music and art may well have a biological origin in that the stimuli resonate with the complexity of the human brain. This speculation is now supported by experiment (*12*) as well as being explained by generalized linear response theory (*3*–*5*) and our extension of Wiener's rule to the PCM (*6*).

3.2 Forgetting

Inverse power-law behavior is known to be characteristic of adaptation, learning, and memory. In this research (*13*), we studied a phenomenological model of forgetting based on renewal theory that introduces a new psychophysical concept, chipping—discrete events that chip away at chunks of memory and thereby produce forgetting. The neural mechanism producing these chips is the 1/f-noise that is generically produced in complex neuronal networks.

From the arguments contained in reference 13, we see that not having a finite average chip generation time, that is, $\mu \leq 2$, as depicted in figure 4, has the effect of facilitating the asymptotic adaptation of the complex network of neurons to coherent memory traces since the rate of chip generation vanishes asymptotically. Consequently, we forget in a well-prescribed manner that slows at an ever decreasing rate and ultimately we stop forgetting altogether. The intuitive explanation of this effect is that a complex network, one described by a hyperbolic distribution such as neural networks, generates signals through the interaction of many distinct neurons. The mathematical models of the interactions in such complex networks indicate the existence of phase transitions and the subsequent generation of intermittent statistics. This intermittent behavior appears to be independent of the details of the interaction mechanism and consequently is a good candidate for describing the underlying statistics of forgetting.

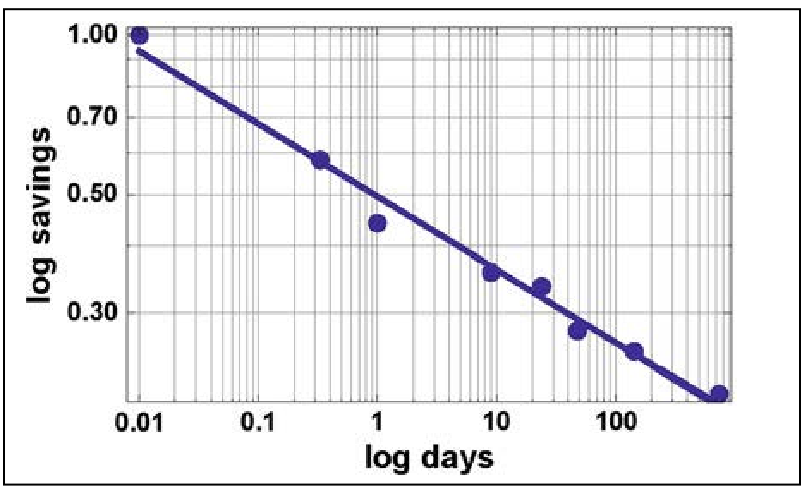

Figure 4. The closed circles are the data from reference 13 for the memory saving function and the line segment is the best mean-square fit to that data. The data are clearly well fit by an inverse power-law function (solid line segment) with a slope of −0.14

The firing of neurons is mediated by biochemistry, so the space-time distribution and activity of chemicals control neuronal firing at the microscopic level and, through cross-level coupling, mediate the statistics of neuron firing. For example, the microscopic activity of ion channel gating has been shown to be fractal, which was found sufficient to account for the fractal-rate firing of neurons. Using this as inspiration, we conjectured that the triggering of the chipping process through *R(t)* may be associated with a PP1-like mechanism, thereby linking the micro- with the macro-dynamics. In fact, a chip may consist of a chemical packet that weakens the synaptic strength associated with memory, but this interpretation awaits a more general theory.

3.3 Decision Making

During this past year we developed a psychophysical model of decision making (*14*) based on the difference between objective clock time and the human brain's perception of time. In this model, the utility function is given by the survival probability, which is shown to be a generalized hyperbolic distribution (asymptotically an inverse power-law), as shown in figure 5. The parameters of the utility function are fit to intertemporal choice model experimental data and decision making is determined to be a 1/f-noise process.

In the present psychophysical model, the index μ provides a measure of dynamic inconsistency, which is to say how soon individuals change their minds with the passage of time, but without new information. The change in the discount rate over time measures irrationality in terms of the deviation of the generalized hyperbolic (Pareto) from the exponential utility function. The empirical estimates using Takahashi's data (*14*) show that irrationality in intertemporal choice is stronger when the outcome is irrelevant to the decision maker, with smaller values of the index μ corresponding to greater irrationality. It is well known that such generalized hyperbolic delay time distributions have diverging mean times for $\mu < 2$, which is the situation for decision making.

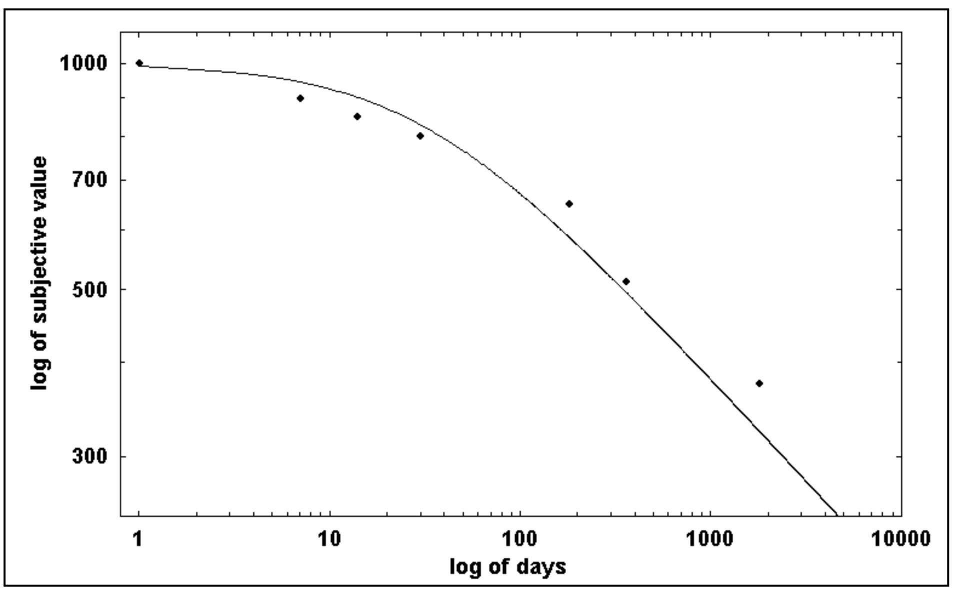

Figure 5. The logarithm of the utility function is plotted as a function of the logarithm of the delay time. The dots are the experimental points for an individual choosing their own rewards with delay and the solid line segment is the fit of a Pareto distribution to the data. The numerical values of the parameters determined by the least-square fit to the data are recorded in reference 14.

We have emphasized that the concept of time is subjective and consequently a strongly impulsive patient may have a perception of time very different from a control individual. It is interesting to observe that stimulant-dependent individuals estimate that a given time interval is larger than the corresponding estimates made by control subjects. Consequently, postponing gratification has a telescoping effect on their discomfort.

Moreover the impulsivity, as measured by the response time in the stochastic intertemporal choice model, is an order of magnitude shorter when the outcome is irrelevant to the decision maker. The interpretation of this latter observation is intriguing, since it appears that a given individual estimates others to be less rational and more impulsive than they are, leading to a number of interesting speculations. Lawmakers, for example, may unconsciously (or even consciously) assume they know what is best for their constituency given their experience and position. Thus, it is also important to emphasize that the response to these stimuli changes according to whether the brain is that of a healthy person or that of a stimulant-dependent subject.

In a military context, this finding, if it is borne out by subsequent experiments, is even more significant. The perceived rationality of others influences such emotional states as trust. A leader may be reluctant to trust subordinates, and therefore not be willing to delegate authority, if their rationality is suspect. It remains for future studies to examine how the irrationality in the choices people make for others can be mitigated.

4. Conclusions

Wiener's rule maintains that a network with high information can organize one with low information. For example, a tightly coupled organization, with rules and policies to cover all contingencies, changes little over time and therefore is low in information. One individual with sufficient information can disrupt such a smoothly running institution, for example, General Billy Mitchell's attempt to separate aviation from the Army and Navy after the First World War because those entities were too traditional and surface-oriented. This separation did occur, but not before Mitchell was court-martialed, convicted, and quickly faded into obscurity, devoid of either influence or importance. But the existence of both Air Force and Naval aviation are due in large part to this one man. This perspective is further elaborated in *Disrupted Networks: From Physics to Climate Change* (*15*).

PCM quantifies Wiener's rule by introducing a measure of information in complex networks, allowing us to compare the level of information in interacting networks. This measure is determined by the power-law index of the hyperbolic distribution and a generalization of linear response theory (LRT) enables us to construct the cross-correlation cube to determine the degree of asymptotic influence one network has on another. In this way, the 1/f variability of stimuli is found to resonate with the human brain, as when we are entranced by music or irritated by a dripping faucet. Thus, there is a mathematical proof of Wiener's rule and its extension to PCM (*5*, *6*).

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